EM375 Project Handout
LAUNCH SPEED vs. STRETCH RATIO

THEORY: If a *linear* spring is stretched by an amount ΔL, the potential energy in the spring is:

$$PE = \frac{1}{2}k\left[\Delta L\right]^2$$

When the spring is released, the potential energy is converted into the kinetic energy of the projectile:

$$\frac{1}{2}k\left[\Delta L\right]^2 = m\frac{V_i^2}{2}$$

This gives the initial launch velocity of the projectile as:

$$V_i = \Delta L\sqrt{\frac{k}{m}}$$

However, this equation CANNOT BE USED FOR THIS PROJECT, because we made three assumptions that do not apply:

1) It assumes that the tubing behaves as a linear elastic material, but the tubing material is not linear.

2) It assumes that the only mass that is accelerated is the projectile mass. However, the launch pouch and the tubing mass are also accelerated.

3) It assumes a 100% efficient transfer of energy from the tubing to the projectile, which is not achievable.

All of these terms will be taken into account.

NONLINEAR MATERIAL: The elastic energy stored in the tubing is the work done in stretching the tube from its initial length to its final length. The elastic potential energy is given by:

$$PE = \int_0^{\Delta L} F dx$$

The force, F, in this equation can be written in terms of the stress in the tubing times the cross sectional area of the tube. Hence:

$$PE = \int_0^{\Delta L} s_n A_O dx$$

where s_n is the normal stress and A_O is the nominal cross sectional area of the tubing.

The strain hardening relationship (from the “rubbers lab”) is:

$$s_n = s_O e_n^a$$

where s_O is a constant with units of psi, a is a dimensionless strain hardening exponent, and e_n is the strain DL/L_O. Recall that in the rubbers lab you determined values for s_O and a. Using the previous two equations, the potential energy stored in the tubing becomes:

$$PE = \frac{A_O s_O}{[a+1]L_O^a}[\Delta L]^{a+1}$$

CORRECTIONS FOR MASS: There are *three* mass terms that have to be considered: The mass of the projectile itself; the mass of the pouch holder; and a fraction of the mass of the tubing. This last term is because some of the tubing (near the holder) is accelerated to the full speed of the projectile, whereas some of the tubing (near the frame) is not accelerated at all.

Let’s see how to deal with the mass of the tubing. We assume that as the tubing moves, the speed of an element of the tube is a linear function of its position down the tube. This means that, for example, if we measure halfway down the tube, the speed will be half of the speed of the water balloon. Hence, the speed, v, of the element is given by $v = V_i \frac{x}{L}$ where x is the position of the element down the tube. The kinetic energy in the element will be:

$$\partial KE = \frac{\partial m_T \times v^2}{2}$$

where m_T is the total mass of the tubing and $\partial m_T = \frac{m_T}{L}\partial x$. Substituting the mass and speed equations into the kinetic energy equation yields:

$$\partial KE = \frac{m_T V_i^2}{2L^3} x^2 \partial x$$

Now let’s integrate to find the total kinetic energy in the rubber band:

$$KE = \int_{x=0}^{L} \partial KE = \int_{x=0}^{L} \frac{m_T V_i^2}{2L^3} x^2 dx = \frac{1}{2}\frac{m_T}{3} V_i^2$$

We compare this result to the “normal” kinetic energy of a mass, and see that the kinetic energy in a band of mass m can be calculated by assuming its effective mass is one-third of its total mass.

The total kinetic energy at launch is therefore:

$$KE = \frac{1}{2}\left(m + \frac{m_T}{3} + m_C\right)V_i^2$$

where m_C is the mass of the pouch.

EFFICIENCY OF ENERGY TRANSFER: We assume that only a certain fraction, η, of the elastic potential energy is converted into kinetic energy. Thus:

$$KE = \eta.PE$$

FINAL RESULT

We combine the previous results and find that the launch speed can be calculated as:

$$V_i = \sqrt{\frac{2.\eta.PE}{\left(m + \frac{m_T}{3} + m_C\right)}}$$

Rather than writing rearranging all the equations for the launch speed, in MathCAD it is probably easier to define several functions

PE(λ)

KE(λ) := η.PE(λ)

and Vi(λ) := (a function with the above equation)